Baseband Data Transmission III

References

– Ideal Nyquist Channel and Raised Cosine Spectrum
  • Chapter 4.5, 4.11, S. Haykin, Communication Systems, Wiley.

– Equalization
  • Chapter 9.11, F. G. Stremler, Communication Systems, Addison Wesley.
The simplest way of satisfying
\[ \sum_{n=\infty}^{\infty} P(f - n/T_b) = T_b \]
is a rectangular function:
\[ p(f) = \begin{cases} \frac{1}{2W} & -W < f < W \\ 0 & |f| > W \end{cases} \]

\[ W = 1 / 2T_b \]
The special value of the bit rate $R_b = 2W$ is called the Nyquist rate, and $W$ is called the Nyquist bandwidth.

This ideal baseband pulse system is called the ideal Nyquist channel

$$p(t) = \frac{\sin(2\pi W t)}{2\pi W t}$$
Ideal Nyquist Channel
In practical situation, it is not easy to achieve it due to

1. The system characteristics of $P(f)$ be flat from $-1/2T$ up to $1/2T$ and zero elsewhere. This is physically unrealizable because of the transitions at the edges.

2. The function $p(t)$ decreases as $1/|t|$ for large $t$, resulting in a slow rate of decay. Therefore, there is practically no margin of error in sampling times in the receiver.
Raised Cosine Spectrum

We may overcome the practical difficulties encountered by increasing the bandwidth of the filter.

Instead of using

\[ p(f) = \begin{cases} 
\frac{1}{2W} & -W < f < W \\
0 & |f| > W 
\end{cases} \]

we use

\[ p(f) + p(f - 2W) + P(f + 2W) = \begin{cases} 
\frac{1}{2W} & -W < f \\
0 & |f| > 1 
\end{cases} \]
Raised Cosine Spectrum

A particular form is a raised cosine filter.
**Raised Cosine Spectrum**

The frequency characteristic consists of a flat amplitude portion and a roll-off portion that has a sinusoidal form. The pulse spectrum \( p(f) \) is specified in terms of a roll off factor \( \alpha \) as follows:

\[
p(f) = \begin{cases} 
\frac{1}{2W} & 0 \leq |f| < f_1 \\
\frac{1}{4W} \left[ 1 - \sin \left( \frac{\pi (|f| - W)}{2W - 2f_1} \right) \right] & f_1 \leq |f| < 2W - f_1 \\
0 & |f| > 2W - f_1
\end{cases}
\]

The frequency parameter \( f_1 \) and bandwidth \( W \) are related by

\[ \alpha = 1 - f_1 / W \]
where $\alpha$ is the rolloff factor. It indicates the excess bandwidth over the ideal solution (Nyquist channel) where $W = 1/2T_b$.

The transmission bandwidth is $(1 + \alpha)W$
Raised Cosine Spectrum

The frequency response of $\alpha$ at 0, 0.5 and 1 are shown in graph below. We observed that $\alpha$ at 1 and 0.5, the function $P(f)$ cutoff gradually as compared with the ideal Nyquist channel and is therefore easier to implement in practice.
The time response $p(t)$ is obtained as

$$p(t) = (\sin c(2Wt))(\frac{\cos(2\pi \alpha Wt)}{1 - 16\alpha^2 W^2 t^2})$$

The function $p(t)$ consists of two parts. The first part is a sinc function that is exactly as Nyquist condition but the second part is depended on $\alpha$. The tails is reduced if $\alpha$ is approaching 1. Thus, it is insensitive to sampling time errors.
Raised Cosine Spectrum
Example:
For $\alpha = 1, (f_1 = 0)$ the system is known as the full-cosine rolloff characteristic.

$$p(f) = \begin{cases} \frac{1}{4W} \left\{ 1 + \cos \left[ \frac{\pi f}{2W} \right] \right\} & 0 < |f| < 2W \\ 0 & |f| > 2W \end{cases}$$

and $p(t) = \frac{\text{sinc}(2Wt)}{1 - 16W^2t^2}$
This time response exhibits two interesting properties:

- At \( t = \pm \frac{T_b}{2} = \pm \frac{1}{4}W \) we have \( p(t) = 0.5 \); that is, the pulse width measured at half amplitude is exactly equal to the bit duration \( T_b \).

- There are zero crossings at \( t = \pm \frac{3T_b}{2}, \pm \frac{5T_b}{2}, \ldots \) in addition to the usual crossings at the sampling times \( t = \pm T_b, \pm 2T_b \).

These two properties are extremely useful in extracting a timing signal from the received signal for the purpose of synchronization. However, the price paid for this desirable property is the use of a channel bandwidth double that required for the ideal Nyquist channel corresponding to \( \alpha = 0 \).
Example:
A sequence of data transmitting at a rate of 33.6 Kbit/s
What is the minimum BW at Nyquist rate.
BW=\( W = \frac{33.6}{2} = 16.8 \text{ KHz} \)
If a 100% rolloff characteristic BW=\( W(1+\alpha) = 33.6 \) KHz
Example

Bandwidth requirement of the T1 system
T1 system: multiplex 24 voice inputs, based on an 8-bit PCM word.

Bandwidth of each voice input \( B \) = 3.1 kHz

Nyquist sampling rate \( f_{\text{Nyquist}} = 2B = 6.2 \text{ kHz} \)

Sampling rate used in telephone system \( f_s = 8 \text{ kHz} \)
With a sampling rate of 8 kHz, each frame of the multiplexed signal occupies a period of 125µs. In particular, it consists of twenty-four 8-bit words, plus a single bit that is added at the end of the frame for the purpose of synchronization.

Hence, each frame consists of a total of 193 bits. Correspondingly, the bit duration is 0.647 µs.
The minimum transmission bandwidth is
\[ \frac{1}{2T_b} = 772\text{kHz} \quad \text{(ideal Nyquist channel)} \]

The transmission bandwidth using full-cosine rolloff characteristics is
\[ W(1 + \alpha) = 2 \times 772\text{kHz} = 1.544\text{MHz} \]
Eye Patterns

This is a simple way to give a measure of how severe the ISI (as well as noise) is. This pattern is generated by overlapping each signal-element.

Example: Binary system

1 0 1 1 0 0 1 1

2T_b
Eye Patterns

Eye pattern is often used to monitoring the performance of baseband signal. If the $S/N$ ratio is high, then the following observations can be made from the eye pattern.

- The best time to sample the received waveform is when the eye opening is largest.
- The maximum distortion and ISI are indicated by the vertical width of the two branches at sampling time.
- The noise margin or immunity to noise is proportional to the width of the eye opening.
- The sensitivity of the system to timing errors is determined by the rate of closure of the eye as the sampling time is varied.
Eye Patterns

Best sampling time

Distortion at sampling time

Margin over noise

Distortion of zero-crossings

Slope = sensitivity to timing error

Time interval over which the received signal can be sampled
Equalization

In preceding sections, raised-cosine filters were used to eliminate ISI. In many systems, however, either the channel characteristics are not known or they vary.

Example
The characteristics of a telephone channel may vary as a function of a particular connection and line used.

It is advantageous in such systems to include a filter that can be adjusted to compensate for imperfect channel transmission characteristics, these filters are called equalizers.
Equalization

Example

\[ x(t) \]

\[ y(t) \]
Equalization

Transversal filter (zero-forcing equalizer)
Equalization

The problem of minimizing ISI is simplified by considering only those signals at correct sample times.

The sampled input to the transversal equalizer is

\[ x(kT) = x_k \]

The output is

\[ x(yT) = y_k \]

For zero ISI, we require that

\[
y_k = \begin{cases} 
1 & k = 0 \\ 
0 & k \neq 0 
\end{cases} \quad \cdots (*)
\]
The output can be expressed as

\[ y_k = \sum_{n=-N}^{N} a_n x_{k-n} \]

There are 2N+1 independent equations in terms of \( a_n \). This limits us to 2N+1 constraints, and therefore (*) must be modified to

\[ y_k = \begin{cases} 
1 & k = 0 \\
0 & k = \pm 1, \pm 2, \ldots, \pm N
\end{cases} \]
Equalization

The 2N+1 equations becomes

\[
\begin{bmatrix}
  x_0 & x_{-1} & \cdots & x_{-N} & \cdots & x_{-2N-1} & x_{-2N} \\
  x_1 & x_0 & \cdots & x_{-N+1} & \cdots & x_{-2N} & x_{-2N+1} \\
  \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots \\
  x_N & x_{N-1} & \cdots & x_0 & \cdots & x_{-N-1} & x_{-N} \\
  \vdots & \vdots & \cdots & \vdots & \ddots & \vdots & \vdots \\
  x_{2N-1} & x_{2N-2} & \cdots & x_{N-1} & \cdots & x_{-2} & x_{-1} \\
  x_{2N} & x_{2N-1} & \cdots & x_N & \cdots & x_{-1} & x_0
\end{bmatrix}
\begin{bmatrix}
  a_{-N} \\
  a_{-N+1} \\
  \vdots \\
  a_0 \\
  \vdots \\
  a_{N-1} \\
  a_N
\end{bmatrix}
= \begin{bmatrix}
  \vdots \\
  \vdots \\
  \vdots \\
  \vdots \\
  \vdots \\
  \vdots \\
  \vdots
\end{bmatrix}
\]
Example
Determine the tap weights of a three-tap, zero-forcing equalizer for the input where
\[ x_{-2} = 0.0, x_{-1} = 0.2, x_0 = 1.0, x_1 = -0.3, x_2 = 0.1, \]
\[ x_k = 0 \text{ for } |k| > 2 \]

The three equations are
\[
\begin{align*}
    a_{-1} + 0.2a_0 &= 0 \\
    -0.3a_{-1} + a_0 + 0.2a_1 &= 1 \\
    0.1a_{-1} - 0.3a_0 + a_1 &= 0
\end{align*}
\]

Solving, we obtain
\[ a_{-1} = -0.1779, a_1 = 0.2847, a_0 = 0.8897 \]
Equalization

The values of the equalized pulse are
\[ y_{-3} = 0.0, y_{-2} = -0.0356, \]
\[ y_{-1} = 0.0, y_0 = 1.0, y_1 = 0.0, \]
\[ y_2 = 0.0036, y_3 = 0.0285 \]

This pulse has the desired zeros to either side of the peak, but ISI has been introduced at sample points farther from the peak.
Equalization
Duobinary Signaling

Intersymbol interference is an undesirable phenomenon that produces a degradation in system performance.

However, by adding intersymbol interference to the transmitted signal in a controlled manner, it is possible to achieve a signaling rate equal to the Nyquist rate of $2W$ symbols per second in a channel of bandwidth $W$ Hz.
Coding and decoding

Consider a binary input sequence \( \{b_k\} \) consisting of uncorrelated binary symbols 1 and 0, each having duration \( T_b \). This sequence is applied to a pulse-amplitude modulator producing a two-level sequence of short pulses (approximating a unit impulse), whose amplitude is

\[
a_k = \begin{cases} 
1 & \text{if symbol } b_k \text{ is 1} \\
-1 & \text{if symbol } b_k \text{ is 0}
\end{cases}
\]

This sequence is applied to a duobinary encoder as shown below:

\[
c_k = a_k + a_{k-1}
\]
One of the effects of the duobinary encoding is to change the input sequence $\{a_k\}$ of uncorrelated two-level pulses into a sequence $\{c_k\}$ of correlated three-level pulses. This correlation between the adjacent pulses may be viewed as introducing intersymbol interference into the transmitted signal in an artificial manner.
Coding and decoding

Example: Consider \( \{b_k\} = 0010110 \) where the first bit is a startup bit.

Encoding:
\[
\begin{align*}
\{b_k\} &: \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \\
\{a_k\} &: \quad -1 \quad -1 \quad +1 \quad -1 \quad +1 \quad +1 \quad -1 \\
\{c_k\} &: \quad -2 \quad 0 \quad 0 \quad 0 \quad 0 \quad +2 \quad 0
\end{align*}
\]

Decoding:
Using the equation \( a_k = c_k - a_{k-1} \), i.e.,
- If \( c_k = +2 \), decide that \( a_k = +1 \).
- If \( c_k = -2 \), decide that \( a_k = -1 \).
- If \( c_k = 0 \), decide opposite of the previous decision.
Duobinary Signaling: Impulse response and frequency spectrum

Let us now examine an equivalent model of the duobinary encoder. The Fourier transfer of a delay can be described as $e^{-2\pi f T_b}$, therefore, the transfer function of the encoder is $H_I(f)$ is

$$H_I(f) = 1 + e^{-j2\pi f T_b}$$

The transfer function of the Nyquist channel is

$$H_N(f) = \begin{cases} 1 & |f| < 1/2T_b \\ 0 & \text{otherwise} \end{cases}$$
Duobinary Signaling: Impulse response and frequency spectrum

The overall equivalent transfer function $H(f)$ of the is then given by

$$H(f) = H_I(f)H_N(f) \quad \text{for} \quad |f| < 1/2T_b$$

$$= (1 + e^{-j2\pi fT_b})$$

$$= (e^{j\pi fT_b} + e^{-j\pi fT_b})e^{-j\pi fT_b}$$

$$= 2e^{-j\pi fT_b} \cos \pi fT_b$$

$H(f)$ has a gradual roll-off to the band edge which can be easily implemented.
Duobinary Signaling: Impulse response and frequency spectrum

The corresponding impulse response \( h(t) \) is found by taking the inverse Fourier transform of \( H(f) \)

\[
h(t) = \frac{\sin(\pi t / T_b)}{\pi t / T_b} + \frac{\sin(\pi (t - T_b) / T_b)}{\pi (t - T_b) / T_b}
\]

\[
= \frac{\sin(\pi t / T_b)}{\pi t / T_b} + \frac{\sin(\pi t / T_b)}{\pi (t - T_b) / T_b}
\]

\[
= T_b^2 \frac{\sin(\pi t / T_b)}{\pi t (t - T_b)}
\]

![Graph showing the impulse response function \( h_1(t) \)]
Notice that there are only two nonzero samples, at $T$-second intervals, give rise to controlled ISI from the adjacent bit. The introduced ISI is eliminated by use of the decoding procedure.